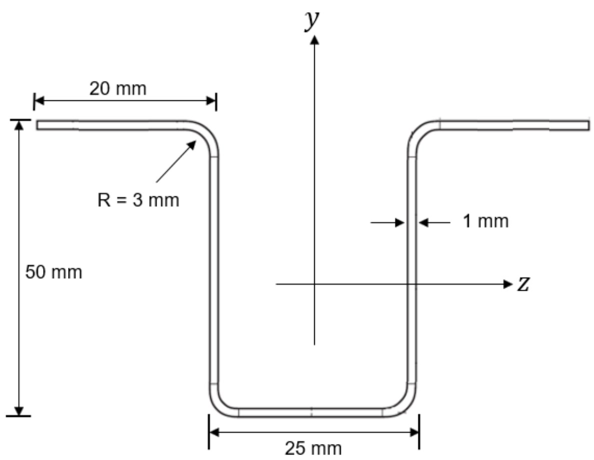
EC3 1-3 2006 CFFD Example 006

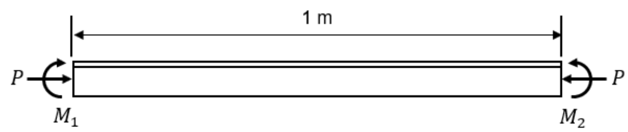
HAT-SECTION MEMBER WITH LIPS UNDER COMBINED COMPRESSION, BENDING, AND SHEAR

EXAMPLE DESCRIPTION

Compression, moment, and shear capacities and demand/capacity ratio are calculated for Hat section with lips at mid-span as shown below. It is simply supported with a length of 1.0 meter.

GEOMETRY, PROPERTIES AND LOADING





Dead: $P = 1000\ N, M_1 = M_2 = 500{,}000\ N - mm$

TECHNICAL FEATURES TESTED

- Axial compressive strength
- Major moment strength
- Shear strength
- Demand/Capacity ratio.

COMPUTER FILE: EC3 1-3 2006 CFFD EX006

Applicable Programs

- SAP2000

RESULTS COMPARISON

Independent results are hand calculated.

CONCLUSION

The results show exact match with independent results.

Benchmarks: SAP2000

Output Parameter	Program	Independent	Percent Difference
Axial - Flexural buckling $N_{b,Rd}$ (N)	31628	31619	0.03%
Axial – Torsional-Flexural buckling $N_{b,Rd}$ (N)	9684	9686	0.02%
Axial – Local & Distortional Buckling $N_{c,Rd}$ (N)	42418	42420	0.00%
Flexure – Lateral-Torsional Buckling $M_{b,Rd}$ $(N-mm)$	----	----	----
Flexure – Local & Distortional Buckling $M_{c,Rd}$ $(N-mm)$	720358	720358	0.00%
Shear $V_{b,Rd}$ (N)	19894	19894	0.00%
D/C Ratio	0.912	0.912	0.00%

HAND CALCULATION

Properties:

Material: $E = 210{,}000\ N/mm^2$, $G = 80{,}770\ N/mm^2$, $f_{yb} = 350\ N/mm^2$

Section:

$h = 25\ mm$, $b = 50\ mm$, $t = 1\ mm$, $c = 20\ mm$, $r = 3\ mm$

$\rightarrow h_p = h - t = 25 - 1 = 24\ mm$

$\rightarrow b_p = b - 1 = 50 - 1 = 49\ mm$

$\rightarrow c_p = c - t/2 = 20 - 1/2 = 19.5\ mm$

Check for the effect of rounding of the corners:

$$\frac{r}{t} = \frac{3}{1} = 3 < 5 \rightarrow OK$$

$$\frac{r}{b_p} = \frac{3}{49} = 0.061 < 0.1 \rightarrow OK$$

Therefore, the effect of rounding of the corners can be neglected in calculation of section properties:

$A_g = 161\ (mm^2)$

$I_y = 34949.677\ (mm^4)$

$I_z = 56585.058\ (mm^4)$

$i_y = 14.734\ (mm)$

$i_z = 18.747\ (mm)$

$W_{el} = 2549.881\ (mm^3)$

$I_t = 53.667\ (mm^4)$

$I_w = 10631863.6\ (mm^6)$

$y_0 = -37.619\ (mm)$

$z_0 = 0.0\ (mm)$

Member: $K_y = K_T = 1.0$ for a pinned-pinned condition

$L_y = L_z = L_T = 1000\ mm$

Member is braced against flexural buckling about z-z axis.

$k_{yy} = k_{zz} = k_{zy} = k_{yz} = 1.0$

Loadings: Dead: $P = 1000\ N, M_1 = M_2 = 500{,}000\ N - mm$

Required strengths: for the section in the middle

$N_{Ed} = P = 1000\ (N)$

$M_{Ed} = M_1 = M_2 = 500{,}000\ (N - mm)$

$V_{Ed} = 0\ (N)$

Member Compression Capacity: the compression capacity is calculated considering the limit states of global buckling, and local and distortional buckling.

1. Local and distortional buckling:
   The effective width method is utilized to calculate the nominal axial strength in consideration of local and distortional buckling with the compressive stress of $f_{yb} = 350\ (N/mm^2)$.

   Check for the applicability of the method as the following conditions are satisfied:

$$\frac{b}{t} = \frac{50}{1} = 50 < 60 \rightarrow OK$$
$$\frac{c}{t} = \frac{20}{1} = 20 < 50 \rightarrow OK$$
$$\frac{h}{t} = \frac{25}{1} = 25 < 500 \rightarrow OK$$
$$\frac{c}{b} = \frac{20}{50} = 0.4 \rightarrow 0.2 < \frac{c}{b} < 0.6 \rightarrow OK$$

   As the section is subjected to uniform compression and both flanges have identical dimensions, they are considered partially stiffened elements with a simple lip edge stiffener and have the same effective properties. The calculation below is only shown for the one of the flanges:

$$\psi = 1$$
$$k_\sigma = 4$$
$$\varepsilon = \sqrt{\frac{235}{f_{yb}[N/mm^2]}} = \sqrt{\frac{235}{350}} = 0.8194$$
$$\bar{\lambda}_{p,b} = \frac{b_p/t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{49/1}{28.4 \times 0.8194\sqrt{4}} = 1.053 > 0.673$$
$$\rho = \frac{\bar{\lambda}_{p,b} - 0.055(3+\psi)}{\bar{\lambda}_{p,b}^2} = \frac{1.053 - 0.055(3+1)}{1.053^2} = 0.751 \leq 1.0$$
$$b_{eff} = \rho b_p = 0.751 \times 49 = 36.82\ (mm)$$
$$b_{e1} = b_{e2} = 0.5 b_{eff} = 0.5 \times 36.82 = 18.41\ (mm)$$

   The lip is considered an outstand (unstiffened) element under uniform compression:

$$\frac{c_p}{b_p} = \frac{19.5}{49} = 0.398 \rightarrow 0.35 < \frac{c_p}{b_p} < 0.6$$
$$k = 0.5 + 0.83\sqrt[3]{\left(\frac{b_{p,c}}{b_p} - 0.35\right)^2} = 0.5 + 0.83\sqrt[3]{\left(\frac{19.5}{49} - 0.35\right)^2} = 0.61$$
$$\bar{\lambda}_{p,c} = \frac{c_p/t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{19.5/1}{28.4 \times 0.8194\sqrt{0.61}} = 1.073 > 0.748$$

$$\rho = \frac{\bar{\lambda}_{p,c} - 0.188}{\bar{\lambda}_{p,b}^2} = \frac{1.073 - 0.188}{1.073^2} = 0.769 \leq 1.0$$

$$c_{eff} = \rho c_p = 0.769 \times 19.5 = 15\ (mm)$$

The stiffener consisting of b_{e2} of the flange and c_{eff} of the lip (Figure 1) is subjected to distortional buckling (b_{e1} of the flange is not affected by distortional buckling and not included in the iterative procedure below):

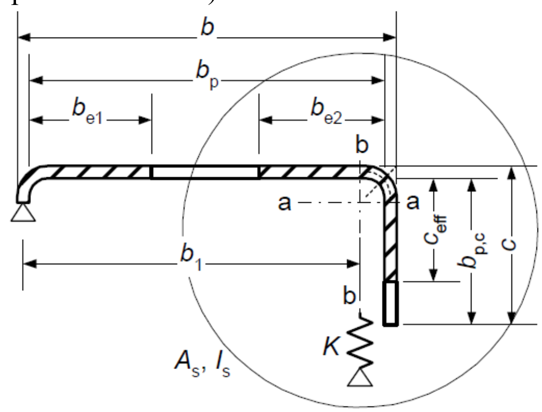


Figure 1 Edge Stiffener (Dubina et al., 2012)

1st iteration:

$$b_1 = b_2 = b_p - \frac{\frac{t b_{e2}^2}{2}}{t\left(b_{e2} + c_{eff}\right)} = 49 - \frac{1 \times \frac{18.41^2}{2}}{1(18.41 + 15)} = 43.93\ (mm)$$

$$A_{s1} = A_{s2} = t\left(b_{e2} + c_{eff}\right) = 1(18.41 + 15) = 33.41\ (mm^2)$$

$$k_{f1} = \frac{A_{s2}}{A_{s1}} = 1$$

$$K = \frac{Et^3}{4(1 - \upsilon^2)} \frac{1}{\left(b_1^2 h_p + b_1^3 + 0.5 b_1 b_2 h_p k_f\right)}$$

$$= \frac{210{,}000 \times 1^3}{4(1 - 0.3^2)} \frac{1}{(43.93^2 \times 24 + 43.93^3 + 0.5 \times 43.93 \times 43.93 \times 24 \times 1)} = 0.374\ (N/mm^2)$$

$$I_s = \frac{t\left(t^2 b_{e2}^2 + 4 b_{e2} c_{eff}^3 + t^2 b_{e2} c_{eff} + c_{eff}^4\right)}{12\left(b_{e2} + c_{eff}\right)}$$

$$= \frac{1(1^2 \times 18.41^2 + 4 \times 18.41 \times 15^3 + 1^2 \times 18.41 \times 15 + 15^4)}{12(18.41 + 15)} = 745.84 (mm^4)$$

$$\sigma_{cr,s} = \frac{2\sqrt{KEI_s}}{A_s} = \frac{2\sqrt{0.374 \times 210{,}000 \times 745.84}}{33.41} = 458.43\ (N/mm^2)$$

$$\bar{\lambda}_d = \sqrt{f_{yb}/\sigma_{cr,s}} = \sqrt{350/458.43} = 0.874 \rightarrow 0.65 < \bar{\lambda}_d < 1.38$$

$$\chi_d = 1.47 - 0.723 \bar{\lambda}_d = 1.47 - 0.723 \times 0.874 = 0.838$$

Since $\chi_d = 0.838 < 1.0 \rightarrow$ iteration is required.

2nd iteration:

b_{e2} of the flange and c_{eff} of the lip are subjected to reduced stress $\sigma_{com,Ed} = \chi_d f_{yb}/\gamma_{M0}$ such that:

$$\bar{\lambda}_{p,b,red} = \bar{\lambda}_{p,b}\sqrt{\chi_d} = 1.053 \times \sqrt{0.838} = 0.964 > 0.673$$

$$\rho = \frac{\bar{\lambda}_{p,b,red} - 0.055(3+\psi)}{\bar{\lambda}^2_{p,b,red}} = \frac{0.964 - 0.055(3+1)}{0.964^2} = 0.801 \leq 1.0$$

$$b_{e2} = 0.5b_{eff} = 0.5\rho b_p = 0.5 \times 0.801 \times 49 = 19.62\ (mm)$$

$$\bar{\lambda}_{p,c,red} = \bar{\lambda}_{p,c}\sqrt{\chi_d} = 1.073 \times \sqrt{0.838} = 0.982 > 0.748$$

$$\rho = \frac{\bar{\lambda}_{p,c,red} - 0.188}{\bar{\lambda}^2_{p,c,red}} = \frac{0.982 - 0.188}{0.982^2} = 0.823 \leq 1.0$$

$$c_{eff} = \rho c_p = 0.823 \times 19.5 = 16.05\ (mm)$$

$$b_1 = b_2 = b_p - \frac{\frac{tb^2_{e2}}{2}}{t(b_{e2} + c_{eff})} = 49 - \frac{1 \times \frac{19.62^2}{2}}{1(19.62 + 16.05)} = 43.6\ (mm)$$

$$A_{s1} = A_{s2} = t(b_{e2} + c_{eff}) = 1(19.62 + 16.05) = 35.67\ (mm^2)$$

$$k_{f1} = \frac{A_{s2}}{A_{s1}} = 1$$

$$K = \frac{Et^3}{4(1-\upsilon^2)} \frac{1}{(b_1^2 h_p + b_1^3 + 0.5 b_1 b_2 h_p k_f)}$$

$$= \frac{210{,}000 \times 1^3}{4(1-0.3^2)} \frac{1}{(43.6^2 \times 24 + 43.6^3 + 0.5 \times 43.6 \times 43.6 \times 24 \times 1)} = 0.381\ (N/mm^2)$$

$$I_s = \frac{t(t^2 b^2_{e2} + 4b_{e2}c^3_{eff} + t^2 b_{e2} c_{eff} + c^4_{eff})}{12(b_{e2} + c_{eff})}$$

$$= \frac{1(1^2 \times 19.62^2 + 4 \times 19.62 \times 16.05^3 + 1^2 \times 19.62 \times 16.05 + 16.05^4)}{12(19.62 + 16.05)} = 914.3\ (mm^4)$$

$$\sigma_{cr,s} = \frac{2\sqrt{KEI_s}}{A_s} = \frac{2\sqrt{0.381 \times 210{,}000 \times 914.3}}{35.67} = 479.74\ (N/mm^2)$$

$$\bar{\lambda}_d = \sqrt{f_{yb}/\sigma_{cr,s}} = \sqrt{350/479.74} = 0.854 \rightarrow 0.65 < \bar{\lambda}_d < 1.38$$

$$\chi_d = 1.47 - 0.723\bar{\lambda}_d = 1.47 - 0.723 \times 0.854 = 0.852$$

Since $\chi_d = 0.852 \neq 0.838$ from previous iteration, more iterations are carried out and the final iteration gives:

$$\chi_d = 0.851.$$

$$b_{e2} = 19.51\ (mm)$$

$$c_{eff} = 15.95\ (mm)$$

$$b_{e1} = 18.41\ (mm)$$

The web is considered an internal (stiffened) element under uniform compression:

$$\psi = 1$$

$$k_\sigma = 4$$
$$\bar{\lambda}_{p,b} = \frac{h_p/t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{24/1}{28.4 \times 0.8194\sqrt{4}} = 0.516 < 0.673 \to \rho = 1.0$$
$$h_{eff} = \rho h_p = 1.0 \times 24 = 24\ (mm)$$
$$A_{eff} = th_{eff} + 2tb_{e1} + 2\chi_d t\left(b_{e2} + c_{eff}\right)$$
$$= 1 \times 24 + 2 \times 1 \times 18.41 + 2 \times 0.851 \times 1(19.51 + 15.95) = 121.2\ (mm^2)$$
$$A_{eff} = 121.2\ (mm^2) < 161\ (mm^2) = A_g$$
$$\to N_{c,Rd} = \frac{A_{eff} f_{yb}}{\gamma_{M0}} = \frac{121.2 \times 350}{1.0} = 42420\ (N)$$

Because the section is symmetric about y-y axis, its effective properties are also symmetric about y-y axis, resulting in $e_{Ny} = 0 \to \Delta M_{y,Ed} = 0$

$$\bar{z} = \frac{\sum_i A_i z_i}{A} = \frac{2tb_p \frac{b_p}{2} + 2tc_p b_p}{A} = \frac{49 \times 49 + 2 \times 19.5 \times 49}{161} = 26.782\ (mm)$$
$$\bar{z}_{eff} = \frac{\sum_i A_{eff,i} z_i}{A_{eff}} = \frac{2tb_{e1} \frac{b_{e1}}{2} + 2\chi_d t b_{e2}\left(b_p - \frac{b_{e2}}{2}\right) + 2\chi_d t c_{eff} b_p}{A_{eff}}$$
$$= \frac{18.41 \times 18.41 + 2 \times 0.851 \times 19.51\left(49 - \frac{19.51}{2}\right) + 2 \times 0.851 \times 15.95 \times 49}{121.2} = 24.534\ (mm)$$
$$e_{Nz} = \bar{z}_{eff} - \bar{z} = 24.534 - 26.782 = -2.248\ (mm)$$
$$\Delta M_{z,Ed} = N_{Ed} e_{Nz} = 1000 \times (-2.248) = -2248\ (N - mm)$$

2. Global buckling: includes flexural buckling and torsional and flexural-torsional buckling
   i. Flexural buckling:

$$N_{cr,y} = \frac{\pi^2 EI_y}{\left(K_y L_y\right)^2} = \frac{\pi^2(210{,}000)34949.677}{(1.0 \times 1000)^2} = 72437.3\ (N)$$
$$N_{cr,z} = \frac{\pi^2 EI_z}{(K_z L_z)^2} = \frac{\pi^2(210{,}000)56585.058}{(1.0 \times 1000)^2} = 117279.15\ (N)$$
$$\bar{\lambda}_y = \sqrt{\frac{A_{eff} f_{yb}}{N_{cr,y}}} = \sqrt{\frac{121.2 \times 350}{72437.3}} = 0.765$$
$$\bar{\lambda}_z = \sqrt{\frac{A_{eff} f_{yb}}{N_{cr,z}}} = \sqrt{\frac{121.2 \times 350}{117279.15}} = 0.601$$

For Hat section with lips, the buckling curve is b and $\alpha = 0.34$

$$\Phi_y = 0.5\left[1 + \alpha\left(\bar{\lambda}_y - 0.2\right) + \bar{\lambda}_y^2\right] = 0.5[1 + 0.34(0.765 - 0.2) + 0.765^2] = 0.889$$
$$\chi_y = \frac{1}{\Phi_y + \sqrt{\Phi_y^2 - \bar{\lambda}_y^2}} = \frac{1}{0.889 + \sqrt{0.889^2 - 0.765^2}} = 0.746$$

$$N_{by,Rd} = \frac{\chi_y A_{eff} f_{yb}}{\gamma_{M1}} = \frac{0.746 \times 121.2 \times 350}{1.0} = 31619\ (N)$$

$$\Phi_z = 0.5\left[1 + \alpha\left(\bar{\lambda}_z - 0.2\right) + \bar{\lambda}_z^2\right] = 0.5[1 + 0.34(0.601 - 0.2) + 0.601^2] = 0.749$$

$$\chi_z = \frac{1}{\Phi_z + \sqrt{\Phi_z^2 - \bar{\lambda}_z^2}} = \frac{1}{0.749 + \sqrt{0.749^2 - 0.601^2}} = 0.836$$

$$N_{bz,Rd} = \frac{\chi_z A_{eff} f_{yb}}{\gamma_{M1}} = \frac{0.836 \times 121.2 \times 350}{1.0} = 35363\ (N)$$

ii. Torsional and flexural-torsional buckling:

$$i_0 = \sqrt{i_y^2 + i_z^2 + y_0^2 + z_0^2} = \sqrt{14.734^2 + 18.747^2 + 0.0^2 + 37.619^2} = 44.54\ (mm)$$

$$N_{cr,T} = \frac{1}{i_0^2}\left[GI_t + \frac{\pi^2 EI_w}{L_T^2}\right] = \frac{1}{44.54^2}\left[80{,}770 \times 53.667 + \frac{\pi^2 210{,}000 \times 10631863.6}{(1.0 \times 1000)^2}\right] = 13293\ (N)$$

$$\beta = 1 - \frac{y_0^2 + z_0^2}{i_0^2} = 1 - \frac{0.0^2 + 37.619^2}{44.54^2} = 0.287$$

$$N_{cr,TF} = \frac{N_{cr,y}}{2\beta}\left[1 + \frac{N_{cr,T}}{N_{cr,y}} - \sqrt{\left(1 - \frac{N_{cr,T}}{N_{cr,y}}\right)^2 + 4\left(\frac{z_0}{i_0}\right)^2 \frac{N_{cr,T}}{N_{cr,y}}}\right]$$

$$= \frac{72437.3}{2 \times 0.287}\left[1 + \frac{13293}{72437.3} - \sqrt{\left(1 - \frac{13293}{72437.3}\right)^2 + 4\left(\frac{37.619}{44.54}\right)^2 \frac{13293}{72437.3}}\right] = 11689\ (N)$$

As $N_{cr,TF} = 19548.2\ (N) < 20022\ (N) = N_{cr,T}$

$$\rightarrow \bar{\lambda}_T = \sqrt{\frac{A_{eff} f_{yb}}{N_{cr,TF}}} = \sqrt{\frac{121.2 \times 350}{11689}} = 1.905$$

For Hat section with lips, the buckling curve for torsional-flexural buckling is b and $\alpha = 0.34$

$$\Phi_T = 0.5\left[1 + \alpha\left(\bar{\lambda}_T - 0.2\right) + \bar{\lambda}_T^2\right] = 0.5[1 + 0.34(1.905 - 0.2) + 1.905^2] = 2.604$$

$$\chi_T = \frac{1}{\Phi_T + \sqrt{\Phi_T^2 - \bar{\lambda}_T^2}} = \frac{1}{2.604 + \sqrt{2.604^2 - 1.905^2}} = 0.228$$

$$N_{b,Rd} = \frac{\chi_T A_{eff} f_{yb}}{\gamma_{M1}} = \frac{0.228 \times 121.2 \times 350}{1.0} = 9686\ (N)$$

Member Flexural Capacity: the flexural capacity is calculated considering the limit states of lateral-torsional buckling, and local and distortional buckling.

1. Local and distortional buckling:
   The effective width method is utilized to calculate the nominal flexural strength in consideration of local and distortional buckling with the compressive stress in the lips of $f_{yb} = 350\ (N/mm^2)$. As the section is subjected to negative moment about the minor axis, the web

is in tension, the lips are in compression, and the flanges are under stress gradient and considered as partially stiffened element with a simple lip edge stiffener. Since the neutral axis is closer to the lips, the web will reach yield stress in tension before the lips reaching yield stress in compression. The plastic reserves in the tension zone may be utilized until the compressive stress reaches yielding. The stress distribution is taken as bilinear in the tension zone and linear in the compression zone as shown on the right in Figure 1:

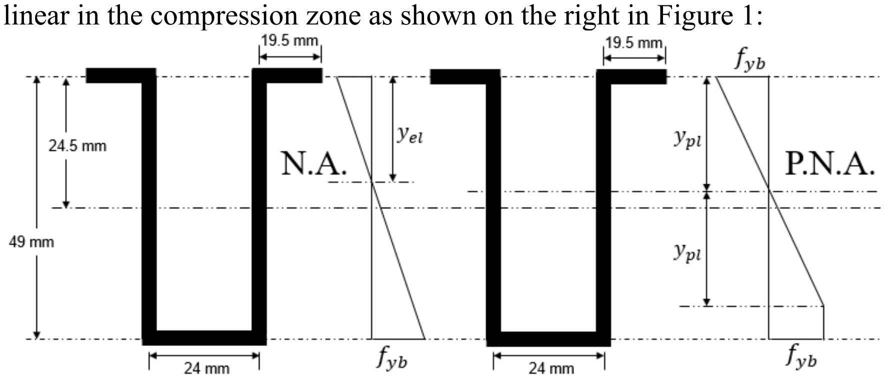


Figure 1: Sections and stress distributions – Left: gross section and linear stress distribution – Right: gross section and bilinear stress distribution.

The location of plastic neutral axis y_{pl} is determined by equating the compressive and tensile forces as follows:

$$\left(2c_p + 2\frac{1}{2}y_{pl}\right)tf_{yb} = \left[2\frac{1}{2}y_{pl} + 2\left(b_p - 2y_{pl}\right) + h_p\right]tf_{yb}$$

$$\rightarrow\ 2c_p = 2\left(b_p - 2y_{pl}\right) + h_p \rightarrow\ y_{pl} = \frac{2b_p - 2c_p + h_p}{4} = \frac{2 \times 49 - 2 \times 19.5 + 24}{2} = 20.75\ (mm)$$

For the flanges, the stress distribution is bilinear, and the stress ratio is allowed to be taken as

$$\psi = -1.0$$

$$k_\sigma = 1.7 - 5\psi + 17.1\psi^2 = 1.7 - 5(-1) + 17.1(-1)^2 = 23.8$$

$$\bar{\lambda}_{p,b} = \frac{b_p/t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{49/1}{28.4 \times 0.8194\sqrt{23.8}} = 0.431 < 0.748 \rightarrow \rho = 1.0$$

$$b_{eff} = \rho y_{pl} = 1.0 \times 20.75 = 20.75\ (mm)$$

$$b_{e1} = 0.4b_{eff} = 0.4 \times 20.75 = 8.3\ (mm)$$

$$b_{e2} = 0.6b_{eff} = 0.6 \times 20.75 = 12.45\ (mm)$$

The lip is considered an outstand (unstiffened) element under uniform compression:

$$\frac{c_p}{b_p} = \frac{19.5}{49} = 0.398 \rightarrow 0.35 < \frac{c_p}{b_p} < 0.6$$

$$k = 0.5 + 0.83\sqrt[3]{\left(\frac{b_{p,c}}{b_p} - 0.35\right)^2} = 0.5 + 0.83\sqrt[3]{\left(\frac{19.5}{49} - 0.35\right)^2} = 0.61$$

$$\bar{\lambda}_{p,c} = \frac{c_p/t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{19.5/1}{28.4 \times 0.8194\sqrt{0.61}} = 1.073 > 0.748$$

$$\rho = \frac{\bar{\lambda}_{p,c} - 0.188}{\bar{\lambda}_{p,b}^2} = \frac{1.073 - 0.188}{1.073^2} = 0.769 \leq 1.0$$

$$c_{eff} = \rho c_p = 0.769 \times 19.5 = 15\ (mm)$$

The stiffener consisting of b_{e1} (instead of b_{e2} because b_{e1} is adjacent to the lip) of the flange and c_{eff} of the lip is subjected to distortional buckling (b_{e2} of the flange is not affected by distortional buckling and not included in the iterative procedure below):

1st iteration:

$$b_1 = b_p - \frac{\frac{tb_{e1}^2}{2}}{t(b_{e1} + c_{eff})} = 49 - \frac{1 \times \frac{8.3^2}{2}}{1(8.3 + 15)} = 47.52\ (mm)$$

$$b_2 = 0$$

$$A_{s1} = A_{s2} = t(b_{e1} + c_{eff}) = 1(8.3 + 15) = 23.3\ (mm^2)$$

$$k_f = \frac{A_{s2}}{A_{s1}} = 1.0$$

$$K = \frac{Et^3}{4(1 - \upsilon^2)} \frac{1}{(b_1^2 h_p + b_1^3 + 0.5 b_1 b_2 h_p k_{f1})}$$

$$= \frac{210{,}000 \times 1^3}{4(1 - 0.3^2)} \frac{1}{(47.52^2 \times 24 + 47.52^3 + 0.5 \times 47.52 \times 47.52 \times 24 \times 1.0)} = 0.306\ (N/mm^2)$$

$$I_s = \frac{t(t^2 b_{e1}^2 + 4 b_{e1} c_{eff}^3 + t^2 b_{e1} c_{eff} + c_{eff}^4)}{12(b_{e1} + c_{eff})}$$

$$= \frac{1(1^2 \times 8.3^2 + 4 \times 8.3 \times 15^3 + 1^2 \times 8.3 \times 15 + 15^4)}{12(8.3 + 15)} = 581.1 (mm^4)$$

$$\sigma_{cr,s} = \frac{2\sqrt{KEI_s}}{A_s} = \frac{2\sqrt{0.306 \times 210{,}000 \times 581.1}}{23.3} = 524.73\ (N/mm^2)$$

$$\bar{\lambda}_d = \sqrt{f_{yb}/\sigma_{cr,s}} = \sqrt{350/524.73} = 0.817 \rightarrow 0.65 < \bar{\lambda}_d < 1.38$$

$$\chi_d = 1.47 - 0.723\bar{\lambda}_d = 1.47 - 0.723 \times 0.817 = 0.879$$

Since $\chi_d = 0.879 < 1.0 \rightarrow$ iteration is required.

2nd iteration:

b_{e1} of the flange and c_{eff} of the lip are subjected to reduced stress $\sigma_{com,Ed} = \chi_d f_{yb}/\gamma_{M0}$:

$$\bar{\lambda}_{p,b,red} = \bar{\lambda}_{p,b}\sqrt{\chi_d} = 0.431 \times \sqrt{0.879} = 0.404 < 0.673 \rightarrow \rho = 1.0$$

$$b_{eff} = \rho y_{pl} = 1.0 \times 20.75 = 20.75\ (mm)$$

$$b_{e1} = 0.4 b_{eff} = 0.4 \times 20.75 = 8.3\ (mm)$$

$$\bar{\lambda}_{p,c,red} = \bar{\lambda}_{p,c}\sqrt{\chi_d} = 1.073 \times \sqrt{0.879} = 1.006 > 0.748$$

$$\rho = \frac{\bar{\lambda}_{p,c,red} - 0.188}{\bar{\lambda}^2_{p,b,red}} = \frac{1.006 - 0.188}{1.006^2} = 0.808 \leq 1.0$$

$$c_{eff} = \rho c_p = 0.808 \times 19.5 = 15.755\ (mm)$$

$$b_1 = b_p - \frac{\frac{tb_{e1}^2}{2}}{t\left(b_{e1} + c_{eff}\right)} = 49 - \frac{1 \times \frac{8.3^2}{2}}{1(8.3 + 15.755)} = 47.57\ (mm)$$

$$A_{s1} = t\left(b_{e1} + c_{eff}\right) = 1(8.3 + 15.755) = 24.055\ (mm^2)$$

$$k_{f1} = \frac{A_{s2}}{A_{s1}} = 1.0$$

$$K = \frac{Et^3}{4(1 - \upsilon^2)} \frac{1}{\left(b_1^2 h_p + b_1^3 + 0.5 b_1 b_2 h_p k_{f1}\right)}$$

$$= \frac{210{,}000 \times 1^3}{4(1 - 0.3^2)} \frac{1}{(47.57^2 \times 24 + 47.57^3 + 0.5 \times 47.57 \times 47.57 \times 24 \times 1.0)} = 0.305\ (N/mm^2)$$

$$I_s = \frac{t\left(t^2 b_{e1}^2 + 4 b_{e1} c_{eff}^3 + t^2 b_{e1} c_{eff} + c_{eff}^4\right)}{12\left(b_{e1} + c_{eff}\right)}$$

$$= \frac{1(1^2 \times 8.3^2 + 4 \times 8.3 \times 15.755^3 + 1^2 \times 8.3 \times 15.755 + 15.755^4)}{12(8.3 + 15.755)} = 663.9\ (mm^4)$$

$$\sigma_{cr,s} = \frac{2\sqrt{KEI_s}}{A_s} = \frac{2\sqrt{0.305 \times 210{,}000 \times 663.9}}{24.055} = 542.2\ (N/mm^2)$$

$$\bar{\lambda}_d = \sqrt{f_{yb}/\sigma_{cr,s}} = \sqrt{350/542.2} = 0.803 \rightarrow 0.65 < \bar{\lambda}_d < 1.38$$

$$\chi_d = 1.47 - 0.723\bar{\lambda}_d = 1.47 - 0.723 \times 0.803 = 0.889$$

Since $\chi_d = 0.889 \neq 0.879$ from previous iteration, more iterations are carried out and the final iteration gives:

$$\chi_d = 0.888 \rightarrow\ t_{eff} = \chi_d t = 0.888 \times 1.0 = 0.888\ (mm)$$

$$b_{e1} = 8.3\ (mm)$$

$$c_{eff} = 15.69\ (mm)$$

$$b_{e2} = 12.45\ (mm)$$

For the linear stress distribution over the mid-height of the section with effective element widths as illustrated in the middle diagram of Figure 2, the section compression (C) and tension (T) forces are:

$$f_1 = f_{yb}\frac{24.5 - b_{e1}}{24.5} = 350\frac{24.5 - 8.3}{24.5} = 231.43\ (N/mm^2)$$

$$C = 2\left[c_{eff} t_{eff} f_{yb} + \frac{1}{2} b_{e1} t_{eff}\left(f_{yb} + f_1\right) + \frac{1}{2}(24.5 - b_{e1}) t f_1\right]$$

$$= 2\left[15.755 \times 0.888 \times 350 + \frac{1}{2} 8.3 \times 0.888(350 + 231.43) + \frac{1}{2}(24.5 - 8.3)1.0 \times 231.43\right]$$

$$= 17828\ (N)$$

$$T = 2 \times \frac{1}{2} 24.5 t f_{yb} + h_p t f_{yb} = 2 \times \frac{1}{2} 24.5 \times 1.0 \times 350 + 24 \times 1.0 \times 350 = 16975 \ (N)$$

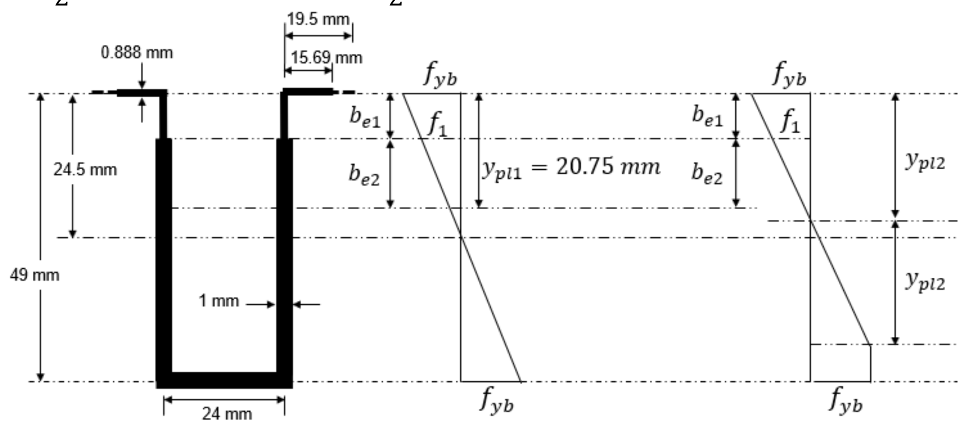


Figure 2: Effective section and stress distributions

As $C = 17828\ (N) > 16975\ (N) = T \rightarrow$ the neutral axis is above the mid-height of the section and the stress distribution is bilinear as demonstrated in right diagram of Figure 2. The neutral axis y_{pl2} is computed by equating the compression and tension forces:

$$C = 2\left[c_{eff} t_{eff} f_{yb} + \frac{1}{2} b_{e1} t_{eff} \left(f_{yb} + f_1\right) + \frac{1}{2}\left(y_{pl2} - b_{e1}\right) t f_1\right] = 2 \times \frac{1}{2} y_{pl2} t f_{yb} + h_p t f_{yb} = T$$

where $f_1 = f_{yb} \frac{y_{pl2} - b_{e1}}{y_{pl2}}$

Solving the equation for $y_{pl2} = 23.91\ (mm)$

$$\rightarrow\ f_1 = f_{yb} \frac{y_{pl2} - b_{e1}}{y_{pl2}} = 350 \frac{23.91 - 8.3}{23.91} = 228.5\ (N/mm^2)$$

The moment contributed by the compression part of the section is:

$$M_C = 2\left[c_{eff} t_{eff} f_{yb} y_{pl2} + \frac{1}{2} b_{e1} t_{eff} f_1 \left(y_{pl2} - \frac{b_{e1}}{2}\right) + \frac{1}{2} b_{e1} t_{eff} \left(f_{yb} - f_1\right)\left(y_{pl2} - \frac{b_{e1}}{3}\right) + \frac{1}{2}\left(y_{pl2} - b_{e1}\right) t f_1 \frac{2}{3}\left(y_{pl2} - b_{e1}\right)\right]$$

$$= 2\left[15.69 \times 0.888 \times 350 \times 23.91 + 8.3 \times 0.888 \times 228.5 \left(23.91 - \frac{8.3}{2}\right) + \frac{1}{2} 8.3 \times 0.888 (350 - 228.5)\left(23.91 - \frac{8.3}{3}\right)\right.$$
$$\left. + \frac{1}{2}(23.91 - 8.3) 1.0 \times 228.5 \frac{2}{3}(23.91 - 8.3)\right] = 356032\ (N - mm)$$

$$M_T = 2\left[\frac{1}{2} y_{pl2} t f_{yb} \frac{2}{3} y_{pl2} + t\left(49 - 2y_{pl2}\right) f_{yb} \left(y_{pl2} + \frac{49 - 2y_{pl2}}{2}\right) + h_p t f_{yb} \left(49 - y_{pl2}\right)\right]$$

$$= 2\left[\frac{1}{2} 23.91 \times 1.0 \times 350 \frac{2}{3} 23.91 + 1.0(49 - 2 \times 23.91) 350 \left(23.91 + \frac{49 - 2 \times 23.91}{2}\right) + 24 \times 1.0 \times 350 (49 - 23.91)\right]$$
$$= 364326\ (N - mm)$$

$$M_{c,Rd} = M_C + M_T = 356032 + 364326 = 720358\ (N - mm)$$

$$W_{eff,c} = \frac{M_{c,Rd}}{f_{yb}} = \frac{720358}{350} = 2058\ (mm^3)$$

$$I_{eff} = W_{eff,c} y_{pl2} = 2058 \times 23.91 = 49211\ (mm^4)$$

$W_{eff,t} = W_{eff,c} = 2058\ (mm^3)$ because the stress distribution is bilinear on the tension side and the bottom edge is also at yield stress.

2. Lateral-torsional buckling:
   Lateral-torsional buckling is only considered for major axis bending. In this example, there is no moment demand on major axis. Therefore, calculation of member moment capacity in consideration of lateral-torsional buckling is not needed.

Member Shear Capacity:

$$\bar{\lambda}_w = 0.346\frac{s_w}{t}\sqrt{\frac{f_{yb}}{E}} = 0.346\frac{49}{1}\sqrt{\frac{350}{210000}} = 0.692 < 0.83$$

$$f_{bv} = 0.58 f_{yb} = 0.58 \times 350 = 203\ (N/mm^2)$$

$$V_{b,Rd} = 2\frac{b_p t f_{bv}}{\gamma_{M0}} = 2\frac{49 \times 1 \times 203}{1.0} = 19894\ (N)$$

Combined D/C ratio:

The ratio by Equation 6.36 in Eurocode 3 1-3 2006 would provide the largest D/C ratio and govern the design

Eq. 6.36 in EC3 1-3:

$$\frac{D}{C} = \left(\frac{N_{Ed}}{N_{b,Rd}}\right)^{0.8} + \left(\frac{M_{y,Ed} + \Delta M_{y,Ed}}{M_{by,Rd}}\right)^{0.8} + \left(\frac{M_{z,Ed} + \Delta M_{z,Ed}}{M_{bz,Rd}}\right)^{0.8}$$

$$= \left(\frac{1000}{9686}\right)^{0.8} + (0)^{0.8} + \left(\frac{500000 + 2248}{720358}\right)^{0.8} = 0.912$$